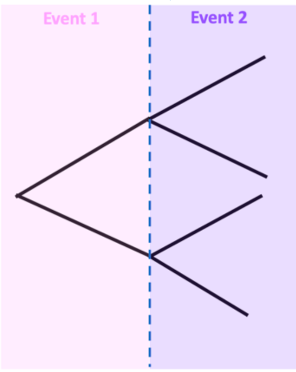
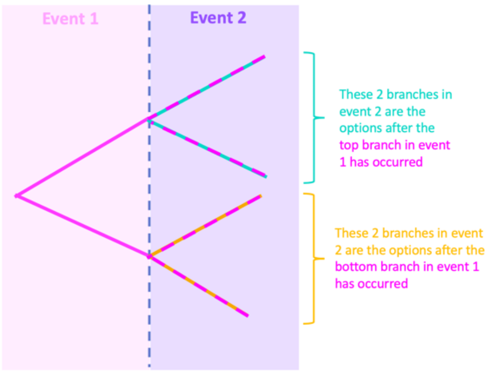
Format and How To Interpret (read this column first):

- The event on the left is the first event
- The event on the right is the next event which happens after event 1 has happened
- Think of the events in your mind as separated by the dashed lines, but in questions these lines won't be drawn for you.



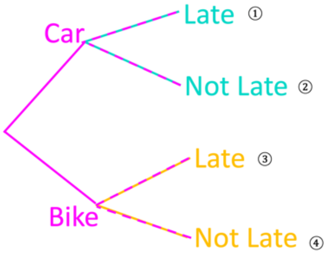
The second event comes after the first. You must understand the following:



For example,

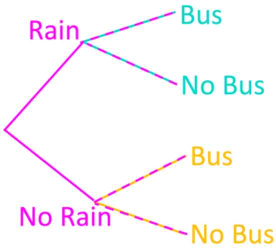
Example 1:

Whether you drive to work by car or bike will affect/influence whether you arrive late or not



Example 2:

Whether it rains or not will affect whether you take the bus to school or not. If the weather is nice you may prefer to walk and not take the bus, right?!



Let's further consider the diagram on the left above

Event 1 branches are bike and cycle and event 2 branches are late and not late.

Our 4 options are:

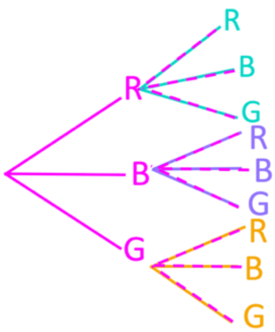
if we drive a car then we can be late ①

if we drive a car then we can also not be late ②

If we ride a bike then we can be late ③

if we ride a bike then we can also not be late ④

We can also have more than 2 events such as 3 events with 3 options like picking 2 balls (2 events) out of a bag which has the 3 colours red, blue and green (3 branches on each)

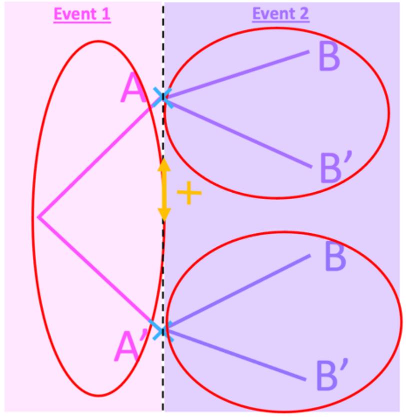


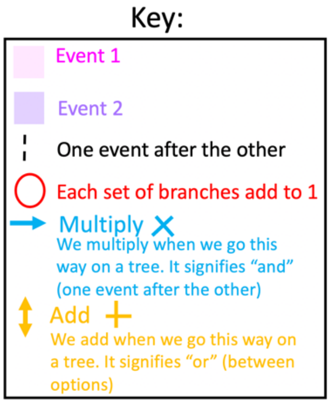
'Rules' (read this column after reading the column on the left):

Don't worry if the following rules don't make sense at first. Read them and then the example underneath will make sure that everything makes sense.

There are 3 rules that must be followed:

- Each set of 2 branches add to 1 (since probabilities add to 1).
  Note: If you have 3 events then a set would consist of 3 branches and each set of 3 probabilities would add to 1 etc
- We multiply when we go across the tree (one event after the other)
- We add when we go down the tree (between options)

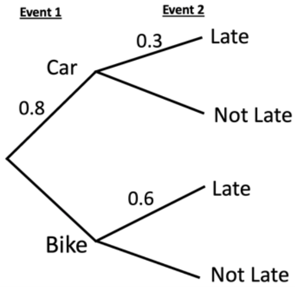




Example:

The following tree diagrams shows the mode of transport taken to work and whether or not you arrive late.

i. Find the probability of taking the car and not arriving late
ii. Find the probability of arriving late to work

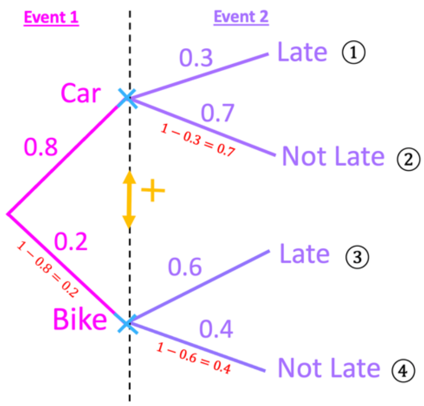


Answer

We can fill in the missing probabilities based on the fact that the probabilities in each pair add to 1 (this is explained under each missing branch)

The symbols (× and +) have been filled in below to help you see when we multiply and when we add

We will colour code the diagram for explanation purpose



i.

This means Car and Not Late

When we say 'and' we multiply (rule is that we multiply when we go across the tree)

$$0.8 \times 0.7 = 0.56$$

ii.

We don't just worry about late, we have to also consider event 1 first since late only occurs after event 1

This means that for Late we must consider Car and Late or Bike and Late

When we say 'and' we multiply (rule:we multiply when we go across the tree)

When we say 'or' we add (rule:we add when go up or down the tree, between options)

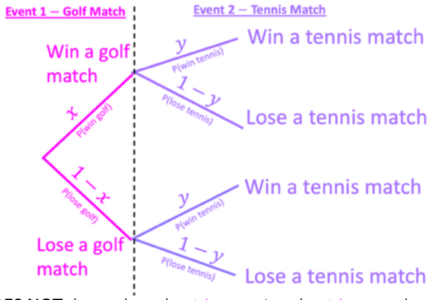
$$CL \text{ or } BL = 0.8 \times 0.3 + 0.2 \times 0.6 = 0.24 + 0.12 = 0.36$$

It is important to understand the difference between independent and dependent events

Independent Events

Note: The explanation below will all make sense once you try the example in the middle of this page, so don't worry if it doesn't make sense at first! You can skip straight to the example if you prefer!

Independent events are when the probability of one event DOES NOT affect the probability of another event. For example, winning a tennis match doesn't depend on winning a golf match. In other words, the outcome of a golf match won't affect the outcome of a tennis match and vice versa. Whether you win or lose the first match which is golf, doesn't affect whether you will win or lose the next match which is the tennis match (unless you're being picky and take the mental side such as confidence from winning the first match into it etc, but of course we don't do this for the purposes of GCSE maths ☺)



The 2nd event **DOES NOT** depend on the 1st event i.e. the 1st event does not affect the probability of the 2nd event.

Important to take away from this:
In both second branches the probabilities will be the same for each pair (both pairs have y and $1-y$). This is because the events are **independent**, so the second event does not change based on what occurred in the first place (the first event).

Example:

There are 5 red pens and 2 blue pens in a pack. Julia takes at random a pen from the pack notes the colour and **puts it back** in the pack.

i. Work out the probability she selects a red pen first and then a blue pen next
ii. Work out the probability she selects two pens the same colour.
iii. Work out the probability she selects two pens of different colour

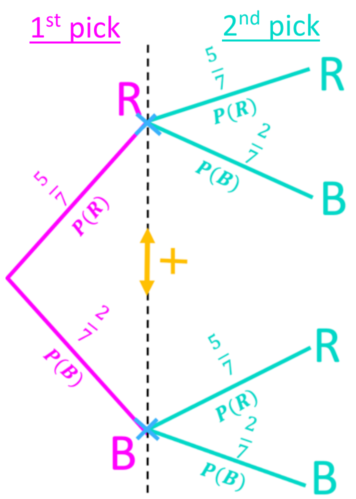
Answer:
We must draw the tree diagram here.
To fill in the probability on each branch we need to use our basic knowledge of probability (see my probability basics cheat sheet if you struggle with this)

$$\text{Probability of an event happening} = \frac{\text{number of times what you're asked for can occur}}{\text{total number of options}}$$

How we get the probabilities for each branch on the tree

We are picking from the following each time
5 red
2 blue
7 total

Hence
Prob of red$=\frac{5}{7}$
Prob of blue$=\frac{2}{7}$



Notice how both pairs of the second branches are the exact same since the first event didn't impact the second event.

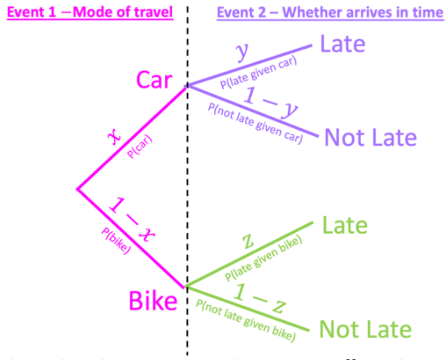
i. This is just R and then B
When we say 'and' we multiply (go across the tree)
$$\frac{5}{7}\times\frac{2}{7}=\frac{10}{49}$$
ii. Same colour can be R and R or B and B
When we say 'and' we multiply (go across the tree)
When we say 'or' we add (go up or down the tree)
$$\text{RR or BB}=\frac{5}{7}\times\frac{5}{7}+\frac{2}{7}\times\frac{2}{7}=\frac{29}{49}$$
iii. Different colours can be R and B or B and R
$$\text{RB or BR}=\frac{5}{7}\times\frac{2}{7}+\frac{2}{7}\times\frac{5}{7}=\frac{20}{49}$$

Independent types rarely come up as they are too easy.

Dependent Events

Note: The explanation below will all make sense once you try the example in the middle of this page, so don't worry if it doesn't make sense at first! You can skip straight to the example if you prefer!

Dependent events are when the probability of one event DOES affect the probability of another event. For example, being late depends on whether travelled by car or bike. The mode of transport will affect how quickly once arrives hence the events are dependent. Notice the word given in the second branches now.



The 2nd events **DOES** depend on the 1st event i.e. the 1st event **affects** the probability of the 2nd event.

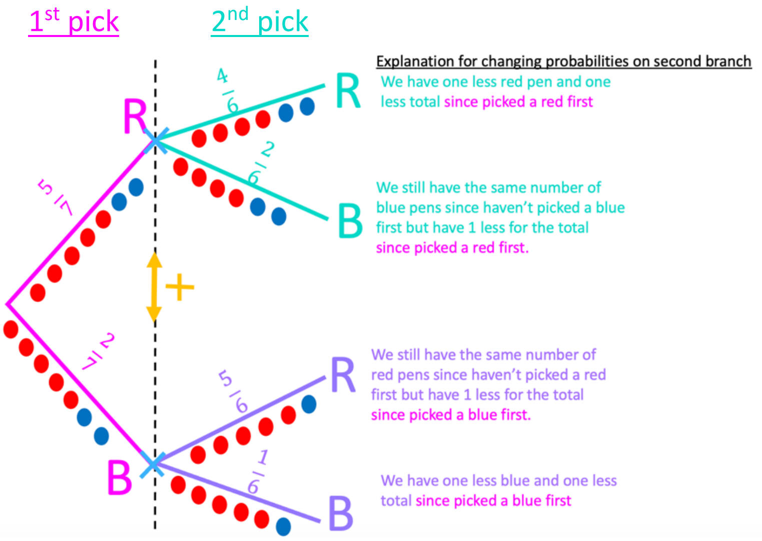
Important to take away from this:
In both second branches the probabilities will NOT be the same for each pair (one pair has y and $1-y$ and the other is equal to z and $1-z$. This is because the events are **dependent**, so the second event CHANGES based on what occurred in the first place (the first event). The difference in colours (purple versus green) has been used to indicate that tey are different.

Example

There are 5 red pens and 2 blue pens in a pack. Julia takes at random a pen from the pack notes the colour and **DOES NOT put it back** in the pack.

i. Work out the probability she selects a red pen first and then a blue pen next
i. Work out the probability she selects two pens the same colour.
ii. Work out the probability she selects two pens of different colour

Answer:
Watch out this time! This question is **harder** since the pen is **not replaced.** We will have to think about the numbers in the numerator and denominator for the second events which is explained below on the right



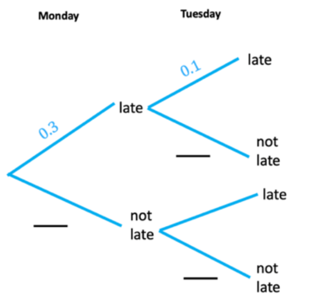
Double check your numbers on the branches are correct by checking, does each set add to 1? Yes they do!

i. This is just R and then B
When we say 'and' we multiply (go across the tree)
$$\frac{5}{7}\times\frac{4}{6}=\frac{20}{42}$$
ii. Same colour can be R and R or B and B
When we say 'and' we multiply (go across the tree)
When we say 'or' we add (go up or down the tree)
$$\text{RR or BB}=\frac{5}{7}\times\frac{4}{6}+\frac{2}{7}\times\frac{1}{6}=\frac{22}{42}=\frac{11}{21}$$
iii. Different colours can be R and B or B and R
$$\text{RB or BB}=\frac{5}{7}\times\frac{2}{6}+\frac{2}{7}\times\frac{5}{6}=\frac{20}{42}=\frac{10}{21}$$

Examples To Try (Independent Events)

Level 1: Bronze

1)Salika travels to school by train every day. The probability that her train will be late on Monday is 0.3 and the probability that he train will be late on Tuesday is 0.1.



i. Complete the probability tree diagram for Monday and Tuesday
ii. Work out the probability her train will be not be late on Monday, but be late on Tuesday
iii. Work out the probability her train will be late on **one day only**
iv. Work out the probability her train will be late on **at least one** of these two days

Level 2: Silver (to draw out)

2)Haseeb is going to play a tennis match and a squash match.
The probability he wins the tennis match is $\frac{7}{10}$
The probability he wins the squash match is $\frac{3}{5}$
Calculate the probability that Haseeb will lose both matches

3)Jo walks to school every day. The probability Jo is late on a Monday is 0.4. The probability Jo is late on a Tuesday is 0.2. Work out the probability that Jo is late on only one of the days.

4)In a supermarket, the probability that John buys fruit is 0.7. In the same supermarket, the probability that John independently buys vegetables is 0.4. Work out the probability that John buys fruit or buys vegetables or buys both.

5)There are only
4 mint biscuits and 1 toffee biscuit in a tin
There are only
5 mint sweets and 3 strawberry sweets in a packet
Michael's mum lets him take one biscuit from the tin and one sweet from the packet
Michael takes a biscuit at random from the tin
He also takes a sweet at random from the packet
Work out the probability that either the biscuit is mint or the sweet is mint, but not both

Level 3: Gold

6)A bag contains 3 black beads, 5 red beads and 2 green beads. Gianna takes a bead at random from the bag, records its colour and **replaces it**. She does this **two more** times. Work out the probability that, of the three beads Gianna takes, exactly two are the same colour.

Level 4: Diamond

7)Two golfers, Smith and Jones, are attempting to qualify for a golf championship. It is estimated that the probability of Jones qualifying is 0.8, and that probability of both Smith and Jones qualifying is 0.6. Given that the probability of Smith qualifying and the probability of Jones qualifying are independent, find the probability that only one of them qualify?

Examples To Try (Dependent Events)

Level 1: Bronze

1)Adam travels to school by car (c) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle. The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car and $\frac{1}{3}$ if he travels by bicycle

i. Find the probability that Adam will travel by car and be late for school
ii. Find the probability that Adam will be late for school

Level 2: Silver (to draw out)

2)In a factory, three machines, J,K and L, are used to make biscuits.
Machine J makes 25% of the biscuits. Machine K makes 45% of the biscuits.
The rest of the biscuits are made by machine L
It is known that 2% of the biscuits made my machine J are broken, 3% of the biscuits made by machine K are broken and 5% of the biscuits made my machine L are broken. A biscuit is selected at random.

i. Calculate the probability that the biscuit is made by machine J and is not broken
ii. Calculate the probability that the biscuit is broken

3)Carolyn has 20 biscuits in a tin. She has
12 plain biscuits, 5 chocolate biscuits and 3 ginger biscuits.
Carolyn takes at random two biscuits from the tin.
Work out the probability that the two biscuits were not the same type

Level 3: Gold

4)There are 9 counters in a bag. There is an even number on 3 of the counters. There is an odd number on 6 of the counters. Three counters are going to be taken at random from the bag. The numbers on the counters will be added together to give the total. Find the probability that the total is an odd number

__With algebra:__

5)There are y black socks and 5 white socks in a drawer. Joshua takes at random two socks from the drawer. The probability that Joshua takes one white sock and one black sock is $\frac{6}{11}$

i. Show that $3y^2 - 28y + 60 = 0$
ii. Find the probability that Joshua takes two black socks

6)There are only green pens and blue pens in a box. There are three more blue pens than green pens in the box. There are more than 12 pens in the box. Simon is going to take at random two pens from the box. The probability that Simon will take two pens of the same colour is $\frac{27}{55}$. Work out the number of green pens in the box.

__With algebra and ratio:__

7)John has an empty box. He puts some red counters and some blue counters into the box. The ratio of the number of red counters to the number of blue counters is 1:4. Linda takes at random 2 counters from the box. The probability that she takes 2 red counters is $\frac{6}{155}$. How many red counters did John put into the box?

__With missing second branches that need to be worked out:__

8)A factory buys 10% of its components from supplier A, 30% from supplier B and the rest from supplier C. It is known that 6 % of the components it buys are faulty.
Of the components bought from supplier A, 9% are faulty and of the components bought from supplier B, 3% are faulty. Find the percentage of components bought from supplier C that are faulty

9)Officials at college are interested in the relationship between participation in interscholastic sports and graduation rate. The following information summarises the probability of several events when a male student is randomly selected.
P(student participates in sports)=0.20
P(student participates in sports and graduates)=0.18
P(student graduates given no participation in sports)=0.80

i. What is the probability that a randomly selected student who participates in sport, graduated?
ii. What is the probability that a randomly selected student graduated?
iii. What is the probability that a randomly selected student participated in sports, given that they graduated?

Level 4: Diamond

__Ongoing events:__

10)Mr Jones has 3 tins of beans and 2 tins of pears. His daughter has removed the labels for a school project, and the tins are identical in appearance. Mr Jones opens tins in turn until he has opened at least 1 tin of beans and at least 1 tin of pears. He does not open any remaining tins

i. Draw a tree diagram to illustrate this situation, labelling each branch with its associated probability
ii. Find the probability that Mr Jones opens exactly 3 tins
iii. It is given that the last tin Mr Jones opens is a tin of pears. Find the probability that he opens exactly 3 tins

__With algebra:__

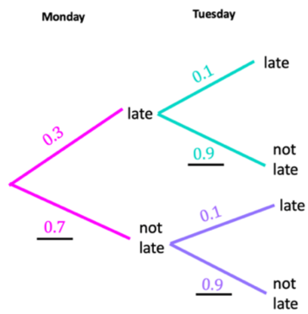
11)There are only red counters, yellow counters and blue counters in a bag. Liam takes at random a counter from the bag. He puts the counter back in the bag. Lethna takes at random a counter from the bag. She puts the counter back in the bag. The probability that both counters are red or that both counters are yellow is $\frac{13}{36}$. The probability that the first counter is red and the second counter is not red is $\frac{1}{4}$. Seb takes at random a counter from the bag. Work out the probability that Seb takes a yellow counter.

12)I have a bag containing some red counters and some blue counters. I draw one counter, and then draw another, having first replaced the first counter. The probability that I draw two red counters is $\frac{1}{9}$.
I have another go, except this time, I do not replace the first counter before drawing the second. The probability that I draw two red counters is $\frac{1}{10}$. Let r be the number of red counters and n be the total number of counters. Find r and n

Answers To Questions In The Left Column:

1)

i.
We use the fact that each colour pair below adds to one to help us fill in the tree diagram



Remember:
- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

i.
$$P(\text{ not late and }\textbf{late}) = 0.7 \times 0.1 = 0.07$$

ii.
$$P(\textbf{ late}\text{ and not late}) = 0.3 \times 0.9 = 0.27$$
$$P(\text{ not late and }\textbf{late}) = 0.7 \times 0.1 = 0.07$$

$$0.27 + 0.07 = 0.34$$

iii.
Way 1:
This means late once or twice

$$P(\textbf{late}\text{ and }\textbf{late}) = 0.3 \times 0.1 = 0.03$$
$$P(\textbf{ late}\text{ and not late}) = 0.3 \times 0.9 = 0.27$$
$$P(\text{ not late and }\textbf{late}) = 0.7 \times 0.1 = 0.07$$

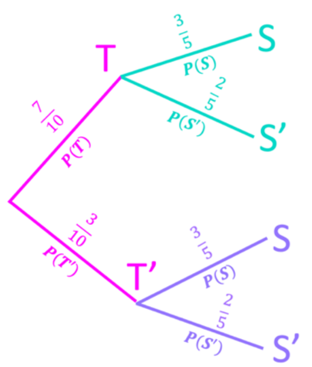
$$0.03 + 0.27 + 0.07 = 0.37$$

Way 2:
Probabilities add to 1
$$1 - P(Not\ Late) = 1 - (0.7 \times 0.9) = 0.37$$

2)

Let P(T) = probability Haseeb wins tennis
Let P(T') = probability Haseeb does not win tennis
Let P(S) = probability Haseeb wins squash
Let P(S') = probability Haseeb does not win squash

Let's draw a tree diagram. We use the fact that colour pair below adds to one to help us fill in the tree diagram



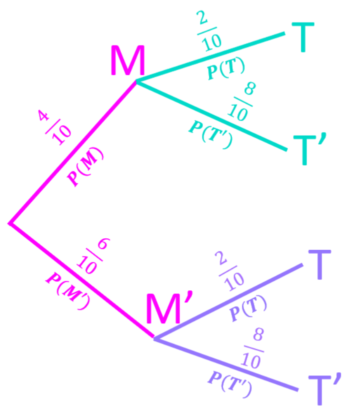
Remember:
- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

P(T' and S') $= \frac{3}{10} \times \frac{2}{5} = \frac{6}{50} = \frac{3}{25}$

3)

Let P(M) = probability Jo is late on Monday
Let P(M') = probability Jo is not late on Monday
Let P(T) = probability Jo is late on Tuesday
Let P(T') = probability Jo is not late on Tuesday

Let's draw a tree diagram. We use the fact that colour pair below adds to one to help us fill in the tree diagram



Remember:

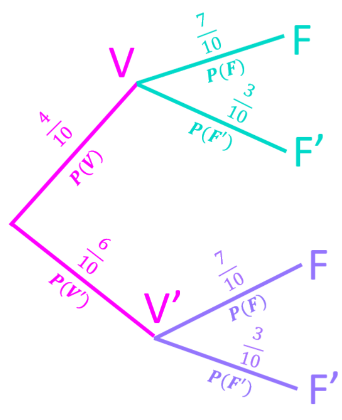
- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

P(M and T') + P(M' and T) = $\left(\frac{4}{10} \times \frac{8}{10}\right) + \left(\frac{6}{10} \times \frac{2}{10}\right) = \frac{11}{25}$

4)

Let P(V) = probability John buys vegetables
Let P(V') = probability John does not buys vegetables
Let P(F) = probability John buys fruit
Let P(F') = probability John does not buys fruit

Let's draw a tree diagram. We use the fact that colour pair below adds to one to help us fill in the tree diagram



Remember:

- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

Instead of working out all of them, we can just do the probability that he does not buy any fruit, and subtract it from 1.

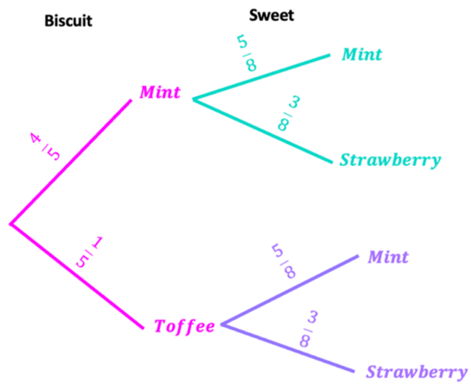
P(V' and F') = $\frac{6}{10} \times \frac{3}{10} = \frac{18}{100} = \frac{9}{50}$

Now subtracting from one

$$1 - \frac{9}{50} = \frac{41}{50}$$

5)

We use the fact that each colour pair below adds to one to help us fill in the tree diagram



Notice how the second branches are dependent on what happened before now and the probability of red in each colour pair on the second branches is not the same unlike for independent events.

Remember:

- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

ii.

$$P(\text{ mint and strawberry}) = \frac{4}{5} \times \frac{3}{8} = \frac{12}{40}$$

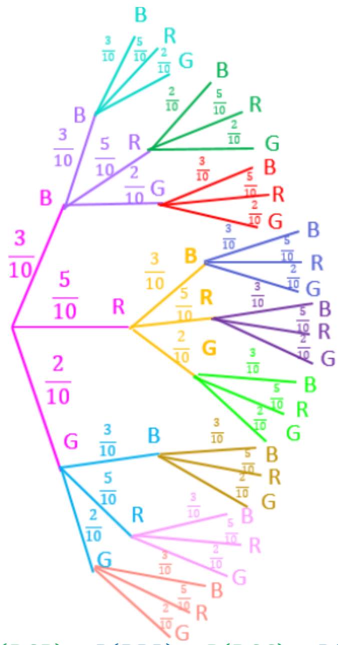
v

$$P(\text{ toffee and mint}) = \frac{1}{5} \times \frac{5}{8} = \frac{5}{40}$$

$$P(\text{ exactly one mint}) = \frac{12}{40} + \frac{5}{40} = \frac{17}{40}$$

6)

Using the facts to draw a tree diagram
For the equation B = Black, R = Red, G = Green



$$P(BBR) + P(BBG) + P(BRB) + P(BGB) + P(BRR) + P(BGG) + P(RRB) + P(RRG) + P(RBB) + P(RGG) + P(RGR) + P(RBR) + P(GGR) + P(GGB) + P(GBB) + P(GRR) + P(GRG) + P(GBG)$$

So this is a lot, but notice how we can group these into groups of 3 by the colours, now doing the math:

$$P(BBR) + P(BBG) + P(BRR) + P(RRG) + P(RGG) + P(GGB)$$

$$= 3\left(\left(\frac{3}{10} \times \frac{3}{10} \times \frac{5}{10}\right) + \left(\frac{3}{10} \times \frac{3}{10} \times \frac{2}{10}\right) + \left(\frac{3}{10} \times \frac{5}{10} \times \frac{5}{10}\right) + \left(\frac{5}{10} \times \frac{5}{10} \times \frac{2}{10}\right) + \left(\frac{5}{10} \times \frac{2}{10} \times \frac{2}{10}\right) + \left(\frac{2}{10} \times \frac{2}{10} \times \frac{3}{10}\right)\right) = \frac{33}{50}$$

7)

Use the fact to draw a tree diagram:
Use the fact that the probability of them both qualifying is 0.6, we can use that to work out probability of smith qualifying:

$$P(Smith) = \frac{0.6}{0.8} = 0.75$$

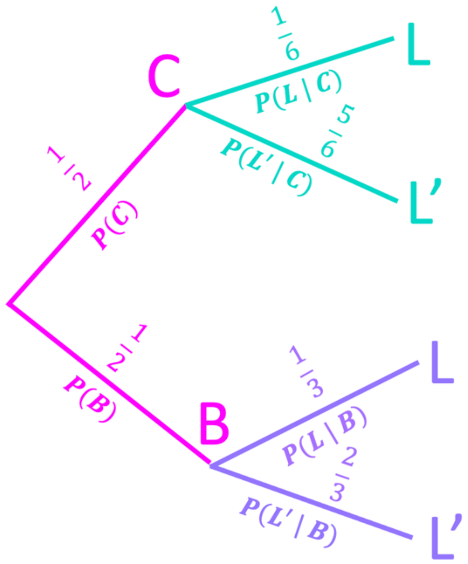
$$P(Qualify\ and\ not\ qualify) + P(not\ qualify\ and\ qualify) = (0.75 \times 0.2) + (0.25 \times 0.8) = 0.35$$

Answers To Questions In The Right Column:

1)

Let P(C) = probability Adam travels by car
Let PB) = probability Adam travels by bike
Let P(L) = probability Adam is late
Let P(L') = probability Adam is <u>not</u> late

Let's draw a tree diagram. We use the fact that colour pair below adds to one to help us fill in the tree diagram



Notice how the second branches are dependent on what happened before now and P(L) in each colour pair is not the same unlike for independent events.

Remember:

- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

i.

$$P(\text{ C and L }) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

ii.

$$P(\text{ L }) = \left(\frac{1}{2} \times \frac{1}{6}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$$

iii.

Here we must use the given formula: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$P(C|L) = \frac{P(C \cap L)}{P(L)} = \frac{\frac{1}{2} \times \frac{1}{6}}{\frac{1}{4}} = \frac{\frac{1}{12}}{\frac{1}{4}} = \frac{4}{12} = \frac{1}{3}$$

iv.

$$LL'L'$$

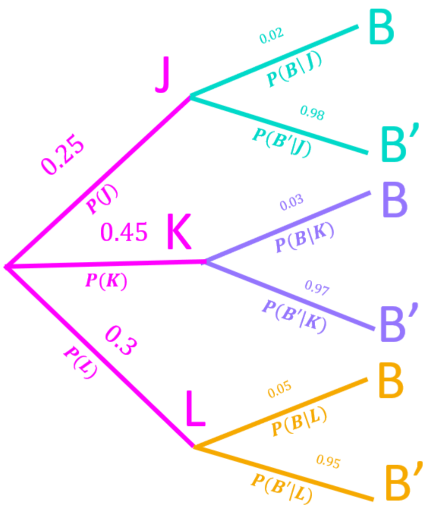
$$\frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{9}{64}$$

There are 3 ways to order this ($L\,L'L'$ or L'LL' or L'L'L)

$$3 \times \frac{9}{64} = \frac{27}{64}$$

2)

Let P(J) = probability of machine J
Let P(K) = probability of machine K
Let P(L) = probability of machine L
Let P(B) = probability broken
Let P(B') = probability not broken



Let's draw a tree diagram. We use the fact that colour pair below adds to one to help us fill in the tree diagram

Notice how the second branches are dependent on what happened before now

Remember:

- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

i.
Reading of the diagram

$$P(\text{J and B}) = 0.02$$

ii.

$$P(J \text{ and } B') = 0.25 \times 0.98 = 0.245$$

iii.

$$P(J \text{ and } B) + P(K \text{ and } B) + P(L \text{ and } B) = (0.25 \times 0.02) + (0.45 \times 0.03) + (0.3 \times 0.05) = 0.0335$$

vii.

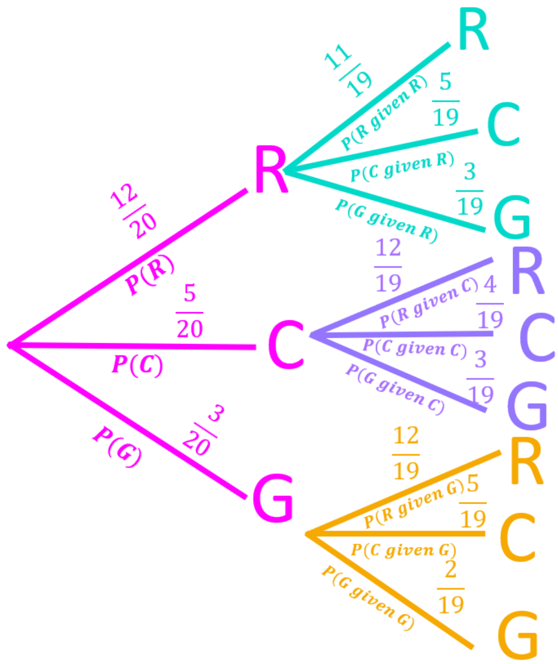
Here we must use the given formula: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$P(J \cap B + \ L \cap B|B) = \frac{P(J \cap B) + P(L \cap B)}{P(B)} = \frac{(0.25)(0.02) + (0.3)(0.05)}{0.0335} = 0.597$$

$$= 64.21\%$$

3)

Let P(P) = probability of a plain biscuit
Let P(C) = probability of a chocolate biscuit
Let P(G) = probability of a ginger biscuit



Let's draw a tree diagram. We use the fact that colour pair below adds to one to help us fill in the tree diagram

Notice how the second branches are dependent on what happened before now

Remember:

- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

We want to work out he probability they are the same and subtract from 1

$$P(\text{P and P}) = \frac{12}{20} \times \frac{11}{19} = \frac{33}{95}$$

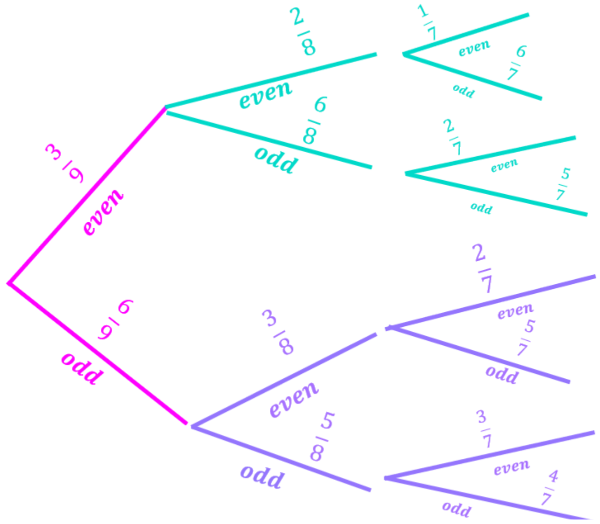
$$P(\text{C and C}) = \frac{5}{20} \times \frac{4}{19} = \frac{1}{19}$$

$$P(\text{G and G}) = \frac{3}{20} \times \frac{2}{19} = \frac{3}{190}$$

$$1 - \left(\frac{33}{95} + \frac{1}{19} + \frac{3}{190}\right) = \frac{111}{190}$$

4)

Use the information to draw a tree diagram



i.

Using what we know from numbers, we know that the sum will be odd:

- if 2 of the terms are even then another is odd
- all three are odd

$$P(EEO)+P(EOE)+P(OEE)+P(OOO)=\left(\frac{3}{9}\times\frac{2}{8}\times\frac{6}{7}\right)+\left(\frac{3}{9}\times\frac{6}{8}\times\frac{2}{7}\right)+\left(\frac{6}{9}\times\frac{3}{8}\times\frac{2}{7}\right)+\left(\frac{6}{9}\times\frac{5}{8}\times\frac{4}{7}\right)=\frac{19}{42}$$

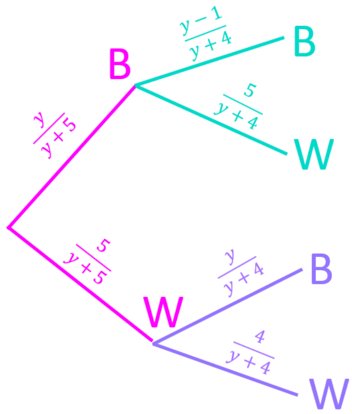
Note: could have done 1 −P(total is even)

5)

Black $= y$
White $= 5$

Let P(W) = probability of getting white sock
Let P(B) = probability of getting a black sock

Let's draw a tree diagram. We use the fact that colour pair below adds to one to help us fill in the tree diagram



Notice how the second branches are dependent on what happened before and hence the numbers are one less once we have chosen

Remember:

- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

P(W and B)+P(B and W)= $\frac{6}{11}$

Subbing in and equating:

$$\left(\frac{y}{y+5}\times\frac{5}{y+4}\right)+\left(\frac{5}{y+5}\times\frac{y}{y+4}\right)=\frac{6}{11}$$

$$\frac{5y}{y^2+9y+20}+\frac{5y}{y^2+9y+20}=\frac{6}{11}$$

$$\frac{10y}{y^2+9y+20}=\frac{6}{11}$$

$$110y=6y^2+54y+120$$

$$0=6y^2-56y+120$$

$$0=3y^2-28y+60$$

ii.
Using equation from i, solve for y:

$$0=3y^2-28y+60$$

$$y=6,\frac{10}{3}$$

We know we can't have a fraction of a sock so y = 6

So now:

$$\text{P(B and B)}=\frac{y}{y+5}\times\frac{y-1}{y+4}=\frac{y(y-1)}{(y+5)(y+4)}$$

Now subbing in y = 6

$$=\frac{6(6-1)}{(6+5)(6+4)}=\frac{3}{11}$$

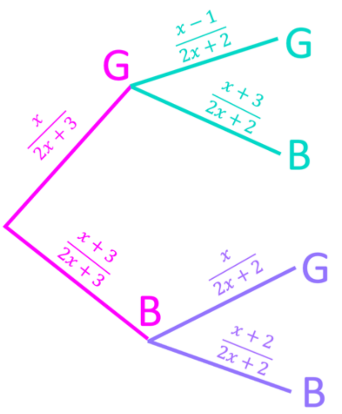
6)

The easiest way to do this is with a tree diagram, but you can certainly do it without.

Call the number of green pens x since we don't know how many

green pens $= x$
blue pens $= x+3$
total $= 2x+3$

Let P(G) = probability of getting a green pen
Let P(B) = probability of getting a blue pen



Remember:

- when we go → we multiply (one event after the other)
- when we go ↓ we add (between option)

P(G and G)+ P(B and B) $= \frac{27}{55}$

$$\left(\frac{x}{2x+3}\times\frac{x-1}{2x+2}\right)+\left(\frac{x+3}{2x+3}\times\frac{x+2}{2x+2}\right)=\frac{27}{55}$$

$$\frac{x^2-x}{(2x+3)(2x+2)}+\frac{x^2+5x+6}{(2x+3)(2x+2)}=\frac{27}{55}$$

$$\frac{2x^2+4x+6}{(2x+3)(2x+2)}=\frac{27}{55}$$

$$55(2x^2+4x+6)=27(2x+3)(2x+2)$$

$$110x^2+220x+330=27(4x^2+10x+6)$$

$$110x^2+220x+330=108x^2+270x+162$$

$$2x^2-50x+168=0$$

$$x^2-25x+84=0$$

$$x=21,4$$

We know the total number must be more the 12:

green pens $= x = 21$

7)

We know the ratio of red to blue is

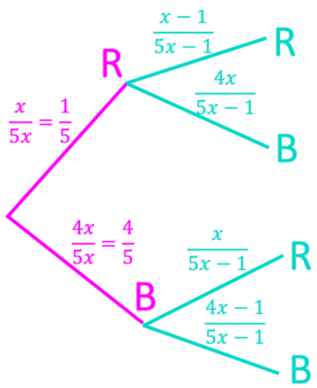
$$1:4$$

Note: The 1:4 does not mean 1 red and 4 blue counters

This in algebra terms, the ratio of red to blue is

$$x:4x$$

Red=x
Blue= $4x$
Total $= 5x$



The probability we get two red is $\frac{6}{155}$

$$\left(\frac{x}{5x}\right)\left(\frac{x-1}{5x-1}\right)=\frac{6}{155}$$

$$\left(\frac{1}{5}\right)\left(\frac{x-1}{5x-1}\right)=\frac{6}{155}$$

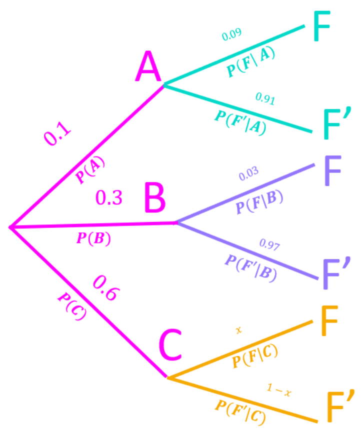
$$\frac{x^2-x}{25x^2-5x}=\frac{6}{155}$$

$$155x^2-155x=150x^2-30x$$

$$5x^2 - 125x = 0$$

$$x = 25$$

8)

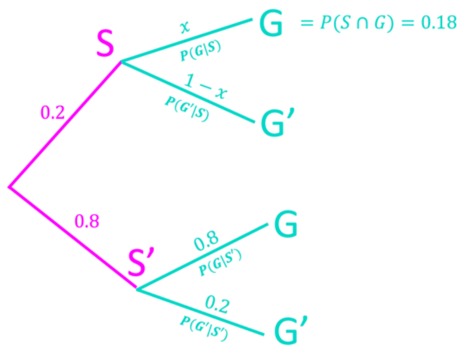


P(F)= 0.06

i.
0.10(0.09)+0.30(0.03)+0.60$x = 0.06$
$0.018 + 0.60x = 0.06$
$0.60x = 0.042$
$x = 0.07$
7%

ii.
If independent $P(B \cap F) = P(B)P(F)$
LHS= $P(B \cap F)$ =0.30(0.03)= 0.009
RHS= $P(B)P(F) = 0.30(0.06) = 0.018$
LHS$\neq RHS$ $\therefore$ not independent

9)



i. Using $P(S \cap G) = 0.18$

$$\frac{0.18}{0.2} = 0.9$$

ii.

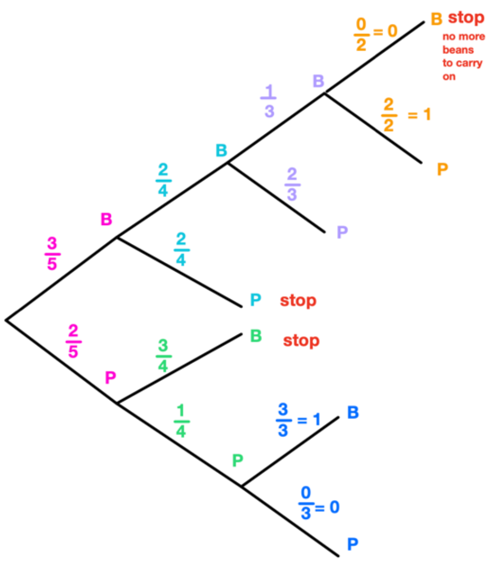
$$P(S \cap G) + P(S' \cap G) = 0.18 + (0.8 \times 0.8)$$
$$= 0.82$$

iii. We know the probability that a student graduated from part ii, use that and the equation:

$$\frac{P(S \cap G)}{P(G)} = \frac{0.18}{0.82} = 0.22$$

10)

i.



ii. BBP or PPB
$\left(\frac{2}{5} \times \frac{2}{4} \times \frac{2}{3}\right) + \left(\frac{2}{5} \times \frac{1}{4} \times 1\right) = \frac{3}{10}$

iii. P(opens 3|last is a pear)= $\frac{opens\ 3 \cap last\ is\ pear}{last\ is\ pear}$

Last is pear options:
BBP or PPB
$\left(\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}\right) + \left(\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}\right) + \left(\frac{3}{5} \times \frac{2}{4}\right) + 0 = \frac{12}{120} + \frac{12}{60} + \frac{6}{20} = \frac{3}{5}$

opens 3 and last is pear options:
BBP or BPP or PBP
$\left(\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}\right) + \left(\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3}\right) + \left(\frac{2}{5} \times \frac{3}{4} \times \frac{1}{3}\right) = \frac{2}{5}$

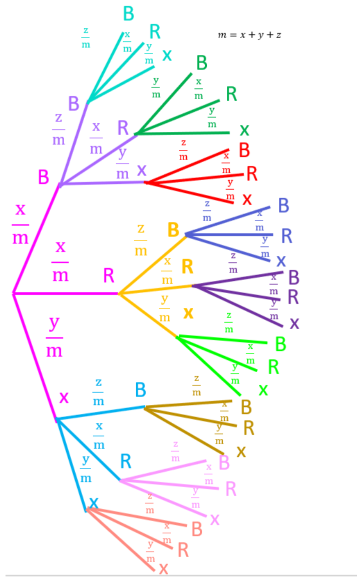
so we get $\frac{\frac{2}{5}}{\frac{3}{5}} = \frac{1}{3}$

11)

Way 1:

First lets draw a tree diagram

P(B) = Blue
P(R) = Red
P(Y) = Yellow



There are two ways we can do this:

Way 1: Equating the probabilities

$$\left(\frac{x}{x+y+z}\right)\left(\frac{x}{x+y+z}\right)+\left(\frac{y}{x+y+z}\right)\left(\frac{y}{x+y+z}\right)=\frac{13}{36}$$

As the denominator is 0:

$$\Rightarrow x^2+y^2=\frac{13}{36}$$

$$\left(\frac{x}{x+y+z}\right)\left(\frac{y+z}{x+y+z}\right)=\frac{1}{4}\Leftrightarrow\left(\frac{x}{1}\right)\left(\frac{1-x}{1}\right)$$

since $x+y+z=1$ (probabilities add to 1)

$$x(1-x)=\frac{1}{4}$$

since $x+y+z=1$

Solve simultaneously

$$x=\frac{1}{2}$$

Now using this to work out y:

$$\left(\frac{1}{2}\right)^2+y^2=\frac{13}{36}$$

$$y^2=\frac{1}{9}$$

$$y=\frac{1}{3}$$

So, the probability he gets a yellow is $\frac{1}{3}$

Way 2:

This time we don't need the tree, we can do it with the symbols alone:

$$r^2+y^2=\frac{13}{36}$$

Using the question:

$$r(1-r)=\frac{1}{4}$$

$$r = \frac{1}{2}$$

Now subbing in a solving for y

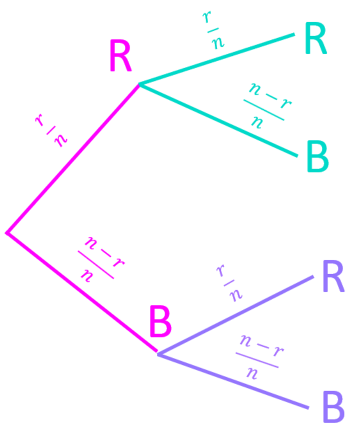
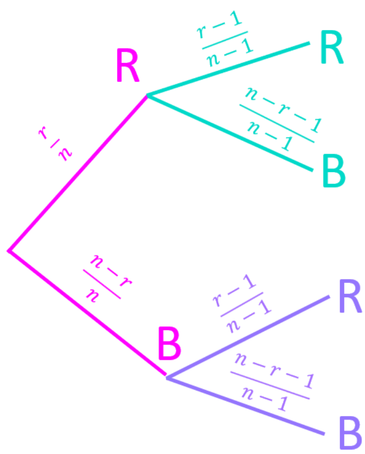
$$\left(\frac{1}{2}\right)^2 + y^2 = \frac{13}{36}$$

$$y^2 = \frac{1}{9}$$

$$y = \frac{1}{3}$$

12)

For this one we may need two trees:
P(R) = Red
P(B) = Blue

With replacement	Without replacement
	
$P(R \cap R)$ $\left(\frac{r}{n}\right)\left(\frac{r}{n}\right) = \frac{r^2}{n^2} = \frac{1}{9}$ $9r^2 = n^2$ $n = 3r$	$P(R \cap R)$ $\left(\frac{r}{n}\right)\left(\frac{r-1}{n-1}\right) = \frac{r^2 - r}{n^2 - n} = \frac{1}{10}$ $10r^2 - 10r = n^2 - n$

So now subbing in the equation from tree one into tree two:

$$10r^2 - 10r = (3r)^2 - (3r)$$

$$10r^2 - 10r = 9r^2 - 3r$$

$$r = 7$$

Subbing into equation one: (n = 3r)

$$n = 21$$